

CS_376 Programming with Abstract Data Types

Coursework 1

Date due: Tuesday, November 3, 2008

Question 1. Use a boolean operation $\text{att}(s, l, c)$, with the intended meaning “student s attended lecture l of course c ”, to formally express the following statements:

- (a) $\text{reg}(s, c) :=$ student s attended course c regularly, that is, missed at most one lecture of course c .
- (b) $\text{model}(s) :=$ student s missed at most two lectures in total.
- (c) $\text{lazy}(s) :=$ student s attended no course regularly.
- (d) No student attended all courses regularly.
- (e) Each course which was attended by at least one student regularly had at least one lecture with full attendance.

Write down a possible signature Σ for your formulas.

[30 marks]

Question 2. For each of the following Σ -formulas decide whether it held in the algebra of our Level 2 Computer Science in 2008/9, or whether it is logically valid.

- (a) $\forall s (\text{model}(s) \rightarrow \neg \text{lazy}(s))$
- (b) $\forall s (\text{reg}(s, \text{cs221}) \rightarrow \neg \text{lazy}(s))$

If you claim that a formula is not logically valid, give a counterexample, that is, a Σ -algebra where the formula does not hold.

If you claim that a formula is logically valid, give a natural deduction proof.

[30 marks]

Question 3. Give natural deduction proofs of the following formulas.

- (a) $P \wedge Q \rightarrow P \vee Q$, in minimal logic;
- (b) $\neg(P \leftrightarrow \neg P)$, in minimal logic;
- (c) $(P \vee Q) \rightarrow (\neg P \rightarrow Q)$, in intuitionistic logic;
- (d) $\neg\forall x P(x) \rightarrow \exists x \neg P(x)$, in classical logic;

[40 marks]

Question 4. (voluntary; not assigned)

Let **Stability** be the formula $\forall x, y (\neg\neg x = y \rightarrow x = y)$. Prove

$$\text{Stability} \vdash_{\text{m}} \neg\neg P \rightarrow P$$

for all formulas P that contain neither \vee nor \exists .

Hint: Use induction on formulas.

Submission

In the lecture on Tuesday, 3rd of November, with a signed Coursework Submission slip attached.